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INDES USER'S GUIDE

MULTISTEP INPUT DESIGN WITH NONLINEAR

ROTORCRAFT MODELING

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TABLE OF CONTENTS

		Page
I.	INTRODUCTION AND OVERVIEW	1
II.	PROGRAM DESCRIPTION	3
	2.1 Program Structure	3
	2.2 Input Design Algorithm	9
III.	PROGRAM INPUT	15
IV.	PROGRAM OUTPUT	43

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I. INTRODUCTION AND OVERVIEW

System identification technology has been used successfully for many vehicles. Because of their large number of degrees of freedom and complex aerodynamic interactions, the rotorcraft have always presented a special challenge to system identification methods. A completely integrated methodology has been developed under this NASA contract to solve this difficult problem. This methodology has also been translated into a user oriented series of computer programs. This volume provides basic guidelines for efficient and effective use of one of these computer programs.

Figure 1.1 shows a schematic flowchart of the overall data processing technique for rotorcraft. The first step in this procedure is state estimation and instrument calibration. This is implemented by the computer program DEKFIS (for Discrete Extended Kalman Filter and Smoother) which implements an extended Kalman filter/smoother using the Friedland-Duffy formulation. Instrument biases and scale factors are estimated at this stage together with any state which is not measured directly. The second step involves estimation of the mathematical model of various forces, moments and interchanges. This is implemented in OSR (Optimal Subset Regression) computer program which uses a regression technique. Accurate estimates of parameters are obtained in the final step. One of two computer programs is used for this purpose. SCIDNT implements the maximum likelihood method for linear systems and NLSCIDNT extends the method to nonlinear rotorcraft models.

Accuracy of parameter estimates may be improved by using flight test inputs based on the input design program, INDES.

This user's manual describes the INDES computer program. The details of the theory and the particular implementation used are given in the final report.*

*Hall, W.S., Gupta, N.K., Hansen, R. and Bohn, J., "State Estimation and Parameter Identification for Rotorcraft," final report on contract NASI-14549, May 1978.

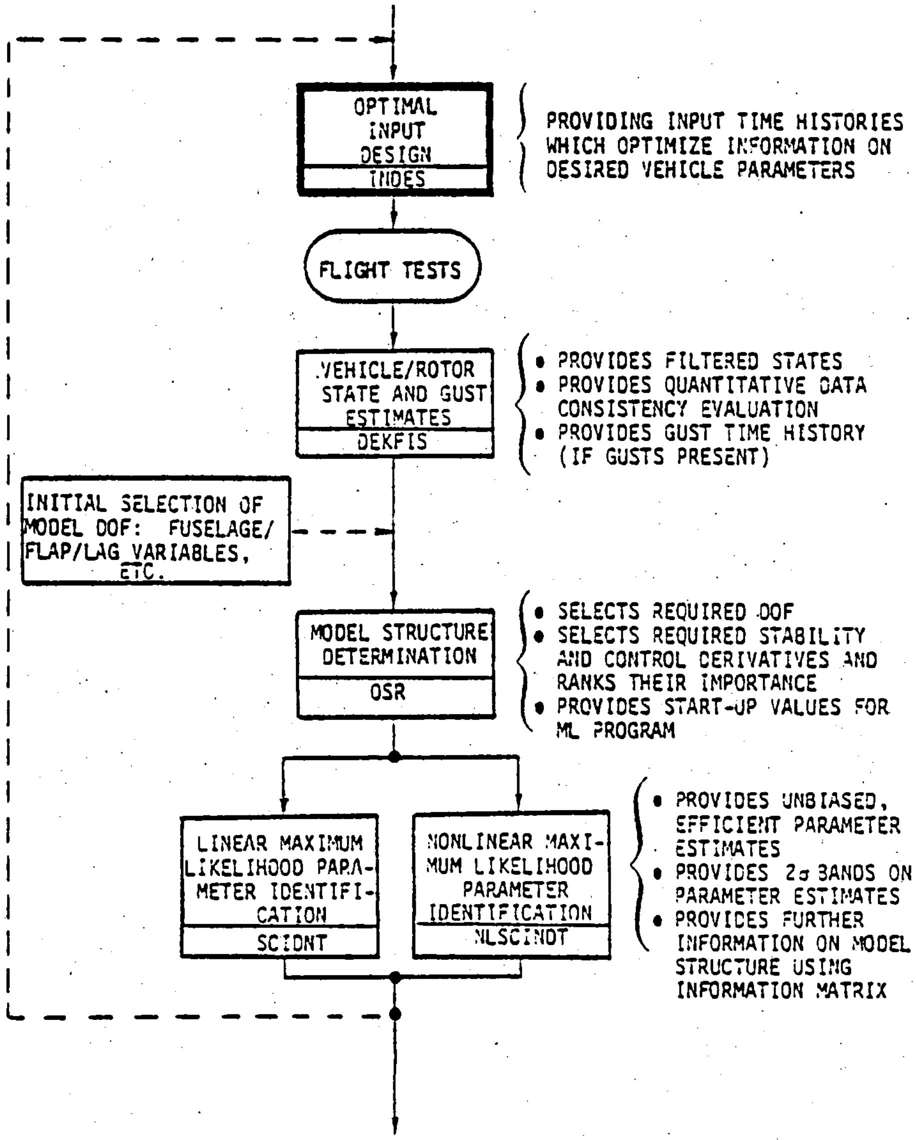


Figure 1.1 Integrated Rotorcraft System Identification Procedure

II. PROGRAM DESCRIPTION

2.1 PROGRAM STRUCTURE

The overall logic of the INDES program is presented in Fig. 2.1. It is intended to give the user a general overview of the program and, therefore, it shows only the major routines with a brief description of their purpose.

The subroutine calling structure is given in Fig. 2.2. The user may find this figure useful in further understanding the program's flow and in constructing an overlay structure if one is needed. The program was developed on a CDC 7600.

The functions of the various subprograms are sketched below:

<u>MAIN</u> is the main routine. It performs no claculations itself, but rather calls three routines--INPUT, INDES, and ØUTPUT--which accomplish the computational and input/output tasks. MAIN's most important function is the dimensioning of the major arrays and setting up most of the labeled commons in the program.

<u>BLKDAT</u> is a block data routine. Its major function is to initialize arrays which contain labels for the measurement, control, and expansion variables for input/output purposes.

<u>INPUT</u> handles all of the program's input, checks for errors in the inputs, and initializes various flags within the program. It also prints this information as a record for the user.

<u>FLAGIT</u> is called by Subroutine INPUT to set a pointer arrays to the state, measurement, and expansion variables which are to be used in the run.

<u>SYMVV1</u> is called by INDES to compute the eigenvalues and eigenvectors of the (symmetric) information matrix. It is optimized for use on symmetric matrices. The eigensystem is calculated by the QR algorithm after the matrix has been put in tridiagonal form by a Householder transformation.

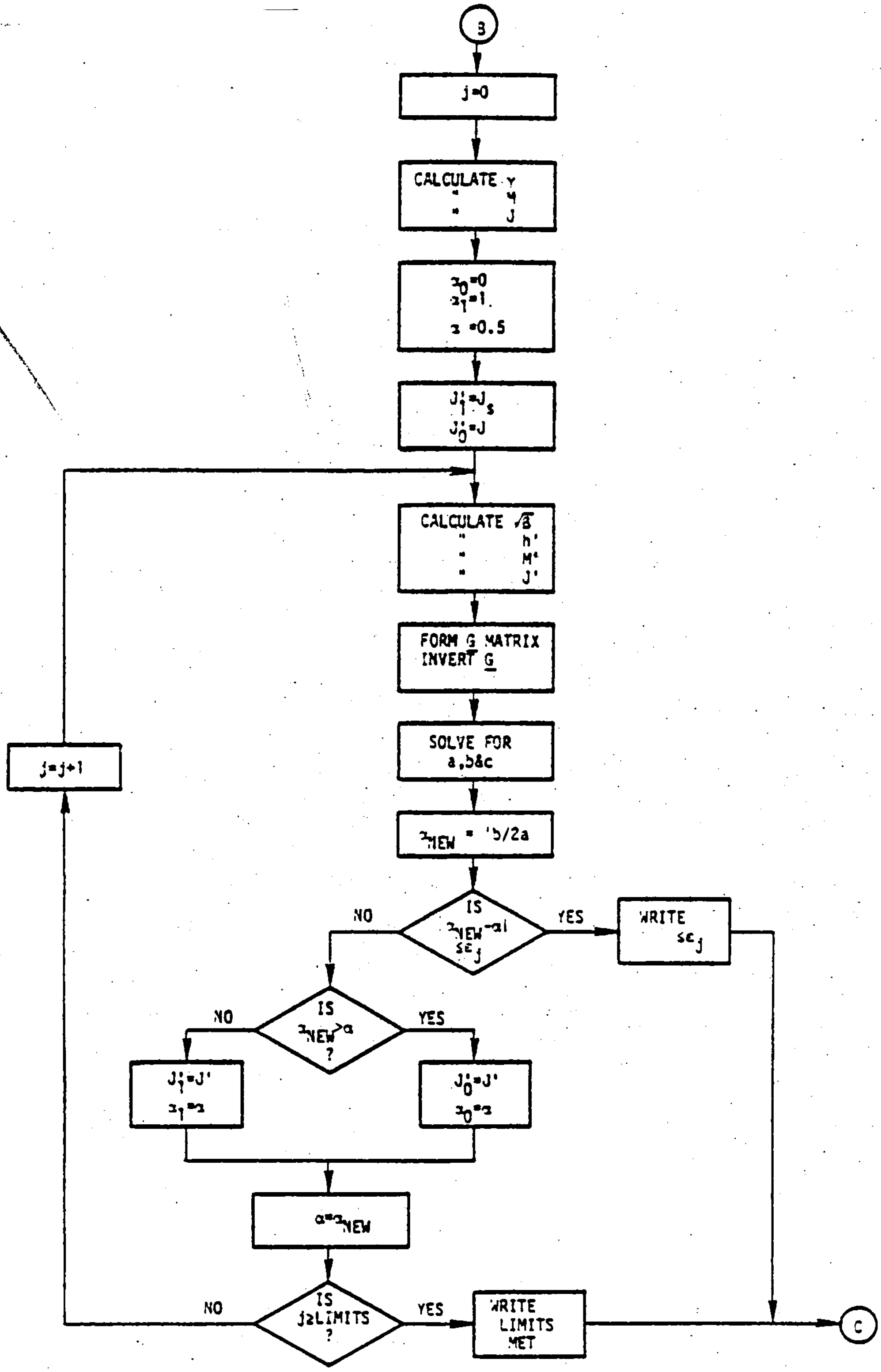
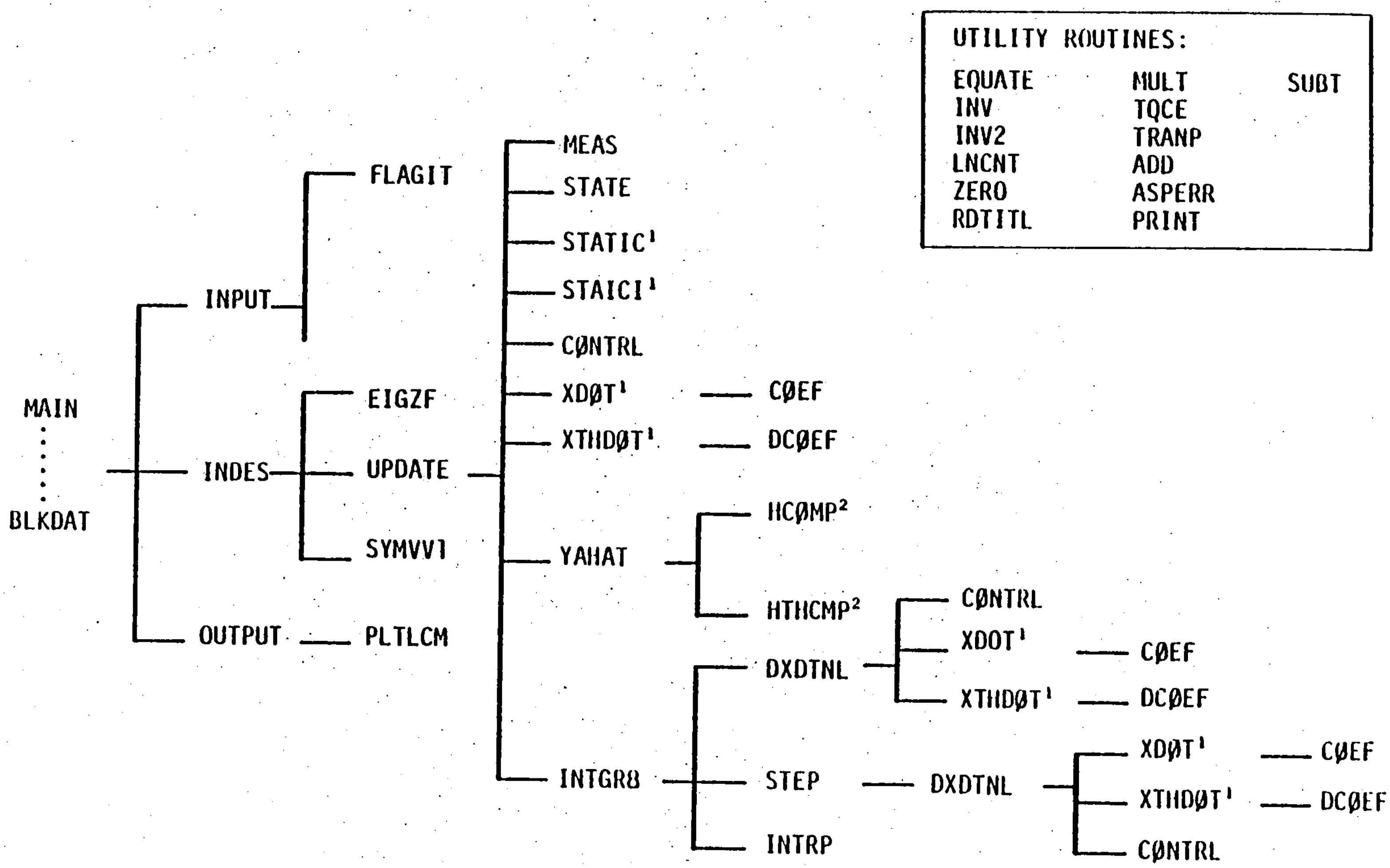


Figure 2.1 INDES Logic



[1]Entry point in STATE
[2]Entry point in MEAS

Figure 2.2 Subroutine Calling Structure

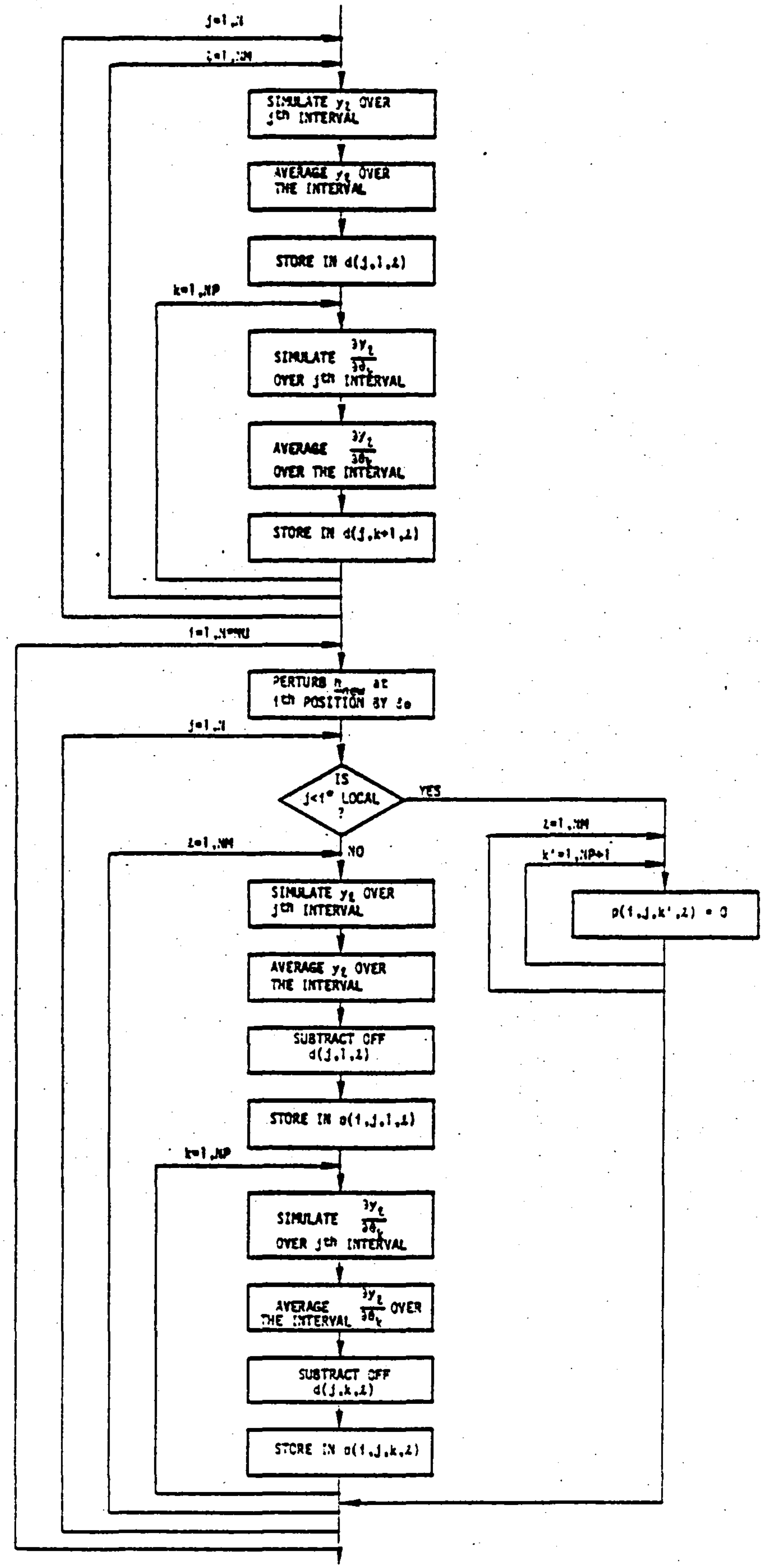


Figure 2.3 Calculation of P Array

UPDATE: (1) propagates the state estimates ($\hat{x}$) and state sensitivities ($\partial\hat{x}/\partial\theta$) with respect to the parameter estimates and computes the measurement estimates ($\hat{y}$) and their sensitivities ($\partial\hat{y}/\partial\hat{\theta}$) by calls to appropriate subroutines; and (2) stores the measurement estimate time histories for use by ØUTPUT.

INTGR8 is called by UPDATE to oversee the integration of the first-order differential equations for $\hat{x}$ and $\partial\hat{x}/\partial\hat{\theta}$ forward one sample time increment. This is accomplished by a variable-order, variable-stepsize, predictor-corrector algorithm (Adams-Krogh Method). The algorithm chooses its own order and stepsize to most efficiently satisfy relative and absolute error bounds on $\hat{x}$ and $\partial x/\partial\theta$ set by the user. The resulting stepsize may be more as well as less than the sample time increment.

STEP is called by INTGR8 to perform the actual integration of the equations. In general, the integration stepsize is not a rational fraction of the sample time interval.

INTRP is called by INTGR8 to interpolate the values of $\hat{x}$, $\partial\hat{x}/\partial\hat{\theta}$ and their time derivatives at the data sampling times.

DXDTNL is called usually by STEP but initially by INTGR8 to obtain the time derivatives of $\hat{x}$ and also $\partial\hat{x}/\partial\hat{\theta}_i$, where $\hat{\theta}_i$ is an identified parameter.

YAHAT is called by UPDATE to evaluate $\hat{y}$ and $\partial\hat{y}/\partial\hat{\theta}$ at the data sample times given $\hat{x}$ and $\partial\hat{x}/\partial\hat{\theta}$ at those times.

STATE has five ENTRY points. A call to STATE initializes variables used elsewhere in the subroutine. STATIC sets the state initial condition estimates, $\hat{x}(0)$. STAICI sets the state sensitivity initial condition estimates, $\partial\hat{x}(0)/\partial\hat{\theta}$. A call to XDØT computes the time derivate of the state estimates, $d\hat{x}/dt$. A call to XTHDØT computes the time derivative of the state sensitivities with respect to a specified parameter, $d(\partial\hat{x}/\partial\hat{\theta}_i)/dt$. Therefore, the vehicle equations of motion appear in this subroutine.

MEAS has three ENTRY points. A call to MEAS evaluates the measurement noise covariance matrix, R. HCØMP computes the measurement estimates, $\hat{y}$. HTHCMP computes the measurement sensitivities with respect to a specified parameter, $\partial\hat{y}/\partial\hat{\theta}_i$. Therefore, the measurement instrument equations appear in this routine.

CØEF evaluates the aerodynamic coefficients required by the vehicle equations of motion in subroutine STATE.

DCØEF evaluates the partial derivatives of the aerodynamic coefficients with respect to each of the identified parameters.

CØNTRL finds the values of the control inputs at any specified time by linear interpolation between the nearest sample time points.

ØUTPUT performs one major task: it uses PLTLCM to plot the final control input time histories and to plot the resulting simulation.

In addition to these major subroutines, there are 13 utility routines of which use is made by several of the above routines.

ADD finds the sum of two matrices.

EQUATE sets one matrix equal to another.

INV finds the inverse and determinant of a matrix.

INV2 finds the inverse and determinant of a matrix.

MULT finds the product of two matrices.

PRNT writes a matrix on the printer.

TRCE computes the trace of a matrix.

RDTITL reads a card containing the run title.

TRANP tranposes a matrix.

SUBT finds the difference of two matrices.

ZERØ sets a matrix equal to the zero matrix.

ASPERR produces a subroutine trace-back in the output when an error in computation is detected.

LNCNT prints the run's title at the top of each page.

PLTLCM produces plots on the printer.

2.2 INPUT DESIGN ALGORITHM

The following steps detail the computational procedures indicated in the various boxes of Fig. 2.1.

2.2.1 The nominal input $h_n(i)$ $i = 1, N*NU$ is given (input)

$$E_n = \sum_{\ell=1}^{N*NU} h_n^2(\ell)$$

$$\left.\begin{aligned} \underline{h}_{new} &= \sqrt{1-k}\ \underline{h}_n \\ \underline{h}_s &= \sqrt{k}\ \underline{h}_n \end{aligned}\right\}$$

- k is input quantity
- $\underline{h}_{new}$, is a (N*NU)*1 vector

$$E_{new} = \sum_{\ell=1}^{N*NU} h_{new}^2(\ell)$$

$$E_s = \sum_{\ell=1}^{N*NU} h_s^2(\ell)$$

where $\underline{h}_{new}$ is a perturbation of $\underline{h}_n$ – scaled by $\sqrt{k}$, typically, k=.01.

2.2.2 h_{new} is composed of NU vectors of length N stacked together. This can be considered as NU separate vectors of length N with i'=1,NU indicating the vector and i_{local}=1,N indicating the component.

2.2.3 y_ℓ for each ℓ=1,NM and $\frac{\delta y_\ell}{\partial \theta_k}$ for each ℓ=1,NM and k=1,NP are simulated by the UPDATE routine in NLS.

In NLSC

$$\underline{YA}(t) = \begin{bmatrix} \underline{y}(t) \\ \frac{\partial \underline{y}(t)}{\partial \theta_1} \\ \vdots \\ \frac{\partial \underline{y}(t)}{\partial \theta_{NP}} \end{bmatrix}$$

Output response

Output sensitivites to θ_1 θ_{NP}

2.2.4 The time histories are to be averaged over each interval so that YA(t) can be expressed as N "discrete" values for each time history

2.2.5 The nominal output response and sensitivity function time histories are stored in the d array.

2.2.6 To peturb h_{new} at the ith position by δ_p (which is an input quantity)

```
NLOCAL = 0
DO i' = 1,NU
DO i_LOCAL = 1,N
i_p = (i'-1)*N + i_LOCAL
h_new_pert(i_p) = h_new(i_p)   (for i_p ≠ i)
h_new_pert(i_p) = h_new(i_p) + δ_p
CONTINUE
CONTINUE
```

$h_{new_{pert}}(i_p) = h_{new}(i_p)$ (for $i_p \neq i$)

$h_{new_{pert}}(i_p) = h_{new}(i_p) + \delta_p$

2.2.7 $Q'(i_1,i_2) = \sum_{j=1}^{N} \sum_{\ell=1}^{NM} p(i_1,j,1,\ell) \cdot p(i_2,j,1,\ell) \cdot Q_1(\ell) + \delta_{i_1,i_2} Q_Z(((L_1-1)/N+1)$

Note: (1) the "1" index in p is for the output time history which is assumed to be stored <u>before</u> the sensitivity function time histories.

(2) $\delta_{i_1 i_2} = \begin{cases} 0 & \text{if } i_1 \neq i_2 \\ 1 & \text{if } i_1 = i_2 \end{cases}$

2.2.8 To calculate M_s

(1) $R_s(j,k,\ell) = \sum_{m=1}^{N*NU} h_s(m)\, p(m,j,k,\ell)$ for all $k>1$

(2) $M_s(k_1-1,k_2-1) = \sum_{j=1}^{N} \sum_{\ell=1}^{NM} (R_s(j,k_1,\ell) \cdot R_s(j,k_2,\ell)/\overline{R}(\ell))$

2.2.9 To calculate $\overline{W}$ (requires inverting M_s)

For <u>Case 1</u>

$$\overline{W} = (M_s^{-1})^T\, W\, M_s^{-1}$$

$$J_s = T_r\{W\, M_s^{-1}\} \quad \text{(W is input)}$$

For <u>Case 2</u>

$$\overline{W} = M_s^{-1}$$

$$J_s = |M_s^{-1}|$$

2.2.10 Calculate V'

$$V'(i_1,i_2) = \sum_{k_1=2}^{NP+1} \sum_{k_2=2}^{NP+1} \left\{ \overline{W}(k_1-1,k_2-1) \cdot \sum_{j=1}^{N} \sum_{\ell=1}^{NM} \frac{p(i_1,j,k_1,\ell)\,p(i_2,j,k_2,\ell)}{\overline{R}(\ell)} \right\}$$

2.2.11 Want to solve the generalized eigenvalue problem:

$$V'h = \lambda Q'h$$

for the largest eigenvalue λ and the corresponding eigenvector h. V' and Q' are both symmetric.

The IMSL routine EIGZF is used which obtains all the eigenvalues for the general problem. For a thorough explanation, see the IMSL documentation, then sort through these for the largest eigenvalue and the corresponding eigenvector.

2.2.12 To calculate λ_{stop}

For <u>Case 1</u> $\lambda_{stop} = \mathrm{Tr}\ \{W\ M_s^{-1}\} + \varepsilon_\lambda$

For <u>Case 2</u> $\lambda_{stop} = NP + \varepsilon_\lambda$

(where ε_λ is input)

2.2.13 $\sqrt{\beta} = -\sqrt{\alpha}\gamma + \sqrt{\alpha\gamma^2 - \alpha + 1}$

$h' = \sqrt{\alpha}\ h_s + \sqrt{\beta}\ h$

2.2.14 $\gamma = h_s^T Q' h$

2.2.15 To calculate M

(1) $R\ (j,k,\ell) = \sum_{m=1}^{N*NU} h(m)p(m,j,k,\ell)$ for all $k>1$

(2) $M\ (k_1-1,k_2-1) = \sum_{j=1}^{N} \sum_{\ell=1}^{NM} (R(j,k_1,\ell) \cdot R(j,k_2,\ell)/\overline{R}(\ell))$

Note that M is a NP*NP matrix.

2.2.16 To calculate J (must invert M)

For <u>Case</u> 1: $J = \mathrm{Tr}\ \{W\ M^{-1}\}$

For <u>Case</u> 2: $J = |M^{-1}|$

2.2.17 To calculate $\sqrt{\beta}$

Solve $\alpha + \beta + 2\sqrt{\alpha\beta}\,\gamma = 1$ for $0 < \sqrt{\beta} < 1$

i.e. $\sqrt{\beta} = -\sqrt{\alpha}\,\gamma + \sqrt{\alpha\gamma^2-\alpha+1}$

2.2.18 $h' = \sqrt{\alpha}\, h_s + \sqrt{\beta}\, h$

2.2.19 To calculate M'

(1) $R_s(j,k,\ell) = \sum_{m=1}^{N*NU} h_s(m)\, p(m,j,k,\ell)$ for all $k>1$

(this should be available from 2.2.8)

(2) $M^*(k_1-1,k_2-1) = \sum_{j=1}^{N} \sum_{\ell=1}^{NM} R_s(j,k_1,\ell) \cdot R(j,k_2,\ell)/\bar{R}(\ell)$

(3) $M' = \alpha M_s + \beta M \quad \sqrt{\alpha\beta}\, M^* + \sqrt{\alpha\beta}\, (M^*)^T$

2.2.20 To calculate J' see 2.2.16 above -- invert M' and use it instead.

2.2.21 $\underline{\underline{G}} = \begin{bmatrix} \alpha_o^2 & \alpha_o & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha_1^2 & \alpha_1 & 1 \end{bmatrix}$

Invert $\underline{\underline{G}}$

2.2.22 $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \underline{\underline{G}}^{-1} \begin{bmatrix} J_o' \\ J' \\ J_1' \end{bmatrix}$

$\alpha_{NEW} = -b/2a$

Other comments: $\left.\begin{array}{l} \text{LIMITJ} \\ \varepsilon_j \end{array}\right\}$ are input on card types 6 and 4 respectively (see Table 3.1)

III. PROGRAM INPUT

3.1 CARD INPUT

Table 3.1 gives the details of how to set up the input data cards. Tables 3.2 through 3.4 give additional information needed to generate the input data and are referred to in Table 3.1 when the information is needed. The parentheses in Table 3.4 indicate parameters of secondary importance which are usually left unused.

A listing of input data is shown in Figure 3.1.

Table 3.1

INDES Input Card Setup

Card Type	Number of Type Cards	Cols.	Format	Name	Description
1	1	1-80	15A4	TITLE	Alphanumeric title for problem
2	1	1-5	I5	N	Number of time steps in the designed control input sequence
		6-10	I5	NU	Number of simultaneous control inputs to be designed
		11-15			
		16-20	I5	NS	Number of states in the assumed model (see card type 15)
		21-25	I5	NM	Number of measurements in the assumed model (see card type 15)
				NP	Number of parameters in the assumed model that are to be identified (i.e. the input is to maximize the identifiability of this many parameters; see card type 16)
3	1	1-20	4I5	NC(I), I=1,NU	Vector indicating which of the eight control inputs in the model are to be used. The value of NC(i) is the appropriate index (see Table 3.2-Control Variables) for the ith control in the simultaneous design.
4	1	1-10	F10.0	T	Total time in maneuver - should be compatible with NN and DELTA in card type 5. I.e. T=(NN-1)*DELTA
		21-30	F10.0	DELTAP	Magnitude of perturbation from nominal input to be used in calculating sensitivities to control step changes.

Table 3.1 (Continued)

Card Type	Number of Type Cards	Cols.	Format	Name	Description
4 Contd.		31-40	F10.0	ε_λ	Allowable error for computing convergence criteria on largest eigenvalue (see Figure 2.1)
		41-50	F10.0	ε_j	Allowable error for computing convergence criteria for J loop (see Figure 2.1)
		51-60	F10.0	EE	Use 0.05 (other options not implemented)
5	1	1-5	I5	NN	Number of time points in continuous time history simulation
		6-15	F10.0	DELTA	Step size in continuous time history simulation
6	1	1-5	I5	NCASE	Case type: NCASE = 1 $J = TR\ \{WM^{-1}\}$ NCASE = 2 $J = \|M^{-1}\|$
		6-10	I5	LIMITI	Max. number of I loop iterations (see Fig. 2.1)
		11-15	I5	LIMITJ	Max. number of J loop iterations (see Fig. 2.1)
		16-20	I5	LIMITO	Use 1 (other options not implemented)
		21-25	I5	KITLIM	Use 3 (other options not implemented)
7	NU- Alternately spaced with an equal number of cards of type 8, i.e. 7-8-7-8....	1-80	8F10.0	HN(i), i_i=1,N	This is the nominal nonlinear control input about which an optimal perturbation input is to be designed. This is typically a constant or zero input setting, but can also be a maneuvering sequence. The optimal control input is the sum of this nominal and the optimum perturbation control input that is designed.
8	NU-see note on card type 7	1-80	8F10.0	HS(i), i=1,N	This is the initial guess for the perturbation (from nominal) control input that is to be optimized

Table 3.1 (Continued)

Card Type	Number of Type Cards	Cols.	Format	Name	Description
9	(k*NP),	1-80	8F10.0	W(i,j), i=1,NP	$\underline{\underline{W}}$ weighting matrix for the identification parameters, 1 row per card (used for Case 1). k=[NP/8] truncated +1
10	1 (or 2 as necessary)	1-80	8F10.0	$\bar{R}$(i), i=1,NM	Diagonal elements of the noise covariance matrix
11	1 (or 2 as necessary)	1-80	8F10.0	Q_1(i), i=1,NM	Parameter weighting terms on outputs in energy constraint expression (see Figure 2.2.7)
12	1	1-20	8F10.0	Q_2(i), i=1,NU	Weighting terms on control inputs in energy constraint expression.
13	1	1-5	I5	IOPRT	Use 4 (other options not implemented)
		6-10	I5	IOPLT	Use 0 (other options not implemented)
		11-15	I5	IOPRT1	Use 0 (other options not implemented)
		16-20	I5	IOPLT1	Use 0 (other options not implemented)
		21-25	I5	IRW	= 1 Writes the control perturbation array (P array) to file IRWUNT after calculating = 2 Reads the control perturbation array (P array) from file IRWUNT. This avoids recalculation = 0 Calculates P array but does not write to file
		26-30	I5	IRWUNT	Output file for storage of control perturbation array (P array). If blank, defaults to Unit 2.

Table 3.1 (Continued)

Card Type	Number of Type Cards	Cols.	Format	Name	Description
14		5	11	IPLØT	= 3 if both printer plots and a tape storing the time history data are to be made = 2 if only the tape is to be made = 1 if only the printer plots are to be made = 0 if neither
		10	11	IPRNT	Sets level of detail of the diagnostic print-out
		12-15	14	INCPR2	= k if $\hat{x}$, $\partial\hat{x}/\partial\hat{\theta}$, $\hat{y}$, $\partial\hat{y}/\partial\hat{\theta}$, and y are to be printed every kth sample point
		17-20	14	INCPR3	Default value is 50. INCPR3 is ignored if IPRNT < 3.
		22-25	14	INCPLT	= k every kth data point is plotted in the printer plots (default = 1)
15		1-10	A2,8X	LTYPE	= STATES, if the card lists the input names of states which are to be integrated = READ, if the card lists the input names of the states which will be looked-up from input time histories = MEASURE, if the card lists the input names of the measurement variables = EXP VARS, if the card lists the input names of the expansion variables = *b, if no more cards of this type are to be read ("b" indicates a blank). Note that only the first two characters are actually read; these words must be <u>left</u> justified.

Table 3.1 (Continued)

Card Type	Number of Type Cards	Cols.	Format	Name	Description
15 Contd.		11-80	14(A3, 2X)	(LL(J), J=1,14)	Input names of state, measurement, or expansion variables as listed in Table 3.2. Note that only the first three characters are actually read; these words must be left justified. Blank words may appear between input names.
16		1	A1	ECHK	= blank, if this card contains parameter information = *, if no more cards of this type are to be read
		2-4	I3	J1	Parameter index (see Table 3.3)
		5	A1	J2	= *, if this parameter is to be identified = blank, if not
		6-11	A6	PLABJ1	Label for the parameter for printout purposes
		12-30	E19.0	PJ1	Initial value of the parameter
		31-32	I2	II(1)	= k, this parameter appears in the polynomial expansion of the kth aero coefficient (see Table 3.4), Ignored if J1 > 400.
		34-50	17L1	II(J), J=2,..,18	Exponents of the expansion variables in the aero coefficient polynomial term containing this parameter
17		1	A1	JA	= T, the label which follows on this card is for time (the independent variable) = Y, the label is for a measurement variable = U, the label is for a control variable = X, the label is for a state variable (only look-up states are affected)

Table 3.1 (Concluded)

Card Type	Number of Type Cards	Cols.	Format	Name	Description
17 Contd.		3-4	I2	J	The index number of the variable. If JA = T, then J is ignored.
		7-54	8A6	NAME	A label of eight 6-character words.

Table 3.2

State, Control, and Measurement Variables

STATE VARIABLES			
INDEX	SYMBOL	DEFINITION	UNITS
1	u	longitudinal component of velocity	ft/sec
2	v	lateral component of velocity	ft/sec
3	w	vertical component of velocity	ft/sec
4	p	body roll rate	rad/sec
5	q	body pitch rate	rad/sec
6	r	body yaw rate	rad/sec
7	ϕ	Euler roll angle	rad
8	θ	Euler pitch angle	rad
9	ψ	Euler yaw angle	rad
10	$\dot{\beta}_o$	coning angular rate	rad/sec
11	$\dot{\beta}_{1c}$	longitudinal flap angular rate	rad/sec
12	$\dot{\beta}_{1s}$	lateral flap angular rate	rad/sec
13	β_o	coning angle	rad
14	β_{1c}	longitudinal flap angle	rad
15	β_{1s}	lateral flap angle	rad
16	$\dot{\zeta}_o$	collective lag angular rate	rad/sec
17	$\dot{\zeta}_{1c}$	longitudinal lag angular rate	rad/sec
18	$\dot{\zeta}_{1s}$	lateral lag angular rate	rad/sec
19	$\delta\dot{\psi}_R$	rotor speed variation	rad/sec
20	ζ_o	collective lag angle	rad
21	ζ_{1c}	longitudinal lag angle	rad
22	ζ_{1s}	lateral lag angle	rad
23	$\delta\psi_R$	rotor angular position variation	rad

Table 3.2 (Continued)

CONTROL VARIABLES			
INDEX	SYMBOL	DEFINITION	UNITS
1	θ_o	collective pitch of blades	rad
2	θ_{1c}	lateral cyclic pitch of blades	rad
3	θ_{1s}	longitudinal cyclic pitch of blades	rad
4	δ_{TR}	pitch of tail rotor blades	rad
5	δ_e	elevator angle	rad
6	δ_a	aileron angle	rad
7	δ_r	rudder angle	rad
8	δ_f	flaperon angle	rad

MEASUREMENT VARIABLES			
INDEX	SYMBOL	DEFINITION	UNITS
1	a_x	longitudinal accelerometer	ft/sec^2
2	a_y	lateral accelerometer	ft/sec^2
3	a_z	vertical accelerometer	ft/sec^2
4	$\dot{p}_m$	roll angular accelerometer	rad/sec^2
5	$\dot{q}_m$	pitch angular accelerometer	rad/sec^2
6	$\dot{r}_m$	yaw angular accelerometer	rad/sec^2
7	p_m	roll rate gyro	rad/sec
8	q_m	pitch rate gyro	rad/sec
9	r_m	yaw rate gyro	rad/sec
10	ϕ_m	roll position gyro	rad
11	θ_m	pitch position gyro	rad
12	ψ_m	yaw position gyro	rad
13	α_m	angle-of-attack vane	rad
14	β_m	sideslip vane	rad
15	V_m	pitot tube	ft/sec

Table 3.2 (Concluded)

MEASUREMENT VARIABLES (Cont'd)			
INDEX	SYMBOL	DEFINITION	UNITS
16	u_m	longitudinal velocity	ft/sec
17	v_m	lateral velocity	ft/sec
18	w_m	vertical velocity	ft/sec
19	X_{R_m}	rotor longitudinal force	lbs
20	Y_{R_m}	rotor lateral force	lbs
21	Z_{R_m}	rotor vertical force	lbs
22	L_{R_m}	rotor roll moment	ft/lbs
23	M_{R_m}	rotor pitch moment	ft/lbs
24	N_{R_m}	rotor yaw moment	ft/lbs
25	β_{1m}	flap angle of blade 1	rad
26	β_{2m}	flap angle of blade 2	rad
27	β_{3m}	flap angle of blade 3	rad
28	β_{4m}	flap angle of blade 4	rad
29	β_{5m}	flap angle of blade 5	rad
30	β_{6m}	flap angle of blade 6	rad
31	β_{7m}	flap angle of blade 7	rad
32	ζ_{1m}	lag angle of blade 1	rad
33	ζ_{2m}	lag angle of blade 2	rad
34	ζ_{3m}	lag angle of blade 3	rad
35	ζ_{4m}	lag angle of blade 4	rad
36	ζ_{5m}	lag angle of blade 5	rad
37	ζ_{6m}	lag angle of blade 6	rad
38	ζ_{7m}	lag angle of blade 7	rad
39	$(\cos \psi_R)_m$	cosine of rotor azimuth	N.D.
40	$(\sin \psi_R)_m$	sine of rotor azimuth	N.D.

Table 3.3

Parameters

INDEX	SYMBOL	DESCRIPTION
1-400		These parameters are available for defining the polynomial expansions of the aerodynamic coefficients
401	$u(0)$	Initial conditions for the state variables
402	$v(0)$	
403	$w(0)$	
404	$p(0)$	
405	$q(0)$	
406	$r(0)$	
407	$\phi(0)$	
408	$\theta(0)$	
409	$\psi(0)$	
410	$\dot{\beta}_o(0)$	
411	$\dot{\beta}_{1c}(0)$	
412	$\dot{\beta}_{1s}(0)$	
413	$\beta_o(0)$	
414	$\beta_{1c}(0)$	
415	$\beta_{1s}(0)$	
416	$\dot{\zeta}_o(0)$	
417	$\dot{\zeta}_{1c}(0)$	
418	$\dot{\zeta}_{1s}(0)$	
419	$\delta\Omega_R(0)$	
420	$\zeta_o(0)$	
421	$\zeta_{1c}(0)$	
422	$\zeta_{1s}(0)$	
423	$\delta\psi_R(0)$	
423-425		Not used.
426	g	Gravitational acceleration (default - 9.80665 m/sec^2)
427	ρ	Air density
428	θ_R	Rotor shaft tilt in X-Z plane, (+) forward

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
429	Ω	Rotor speed
430	R	Rotor radius
431	σ	NC/ R
432	γ	Lock number = $\rho acR^4/I_b$
433	a	2-D lift curve slope
434	m	Mass of vehicle
435	x_{HUB}	Longitudinal distance of rotor hub from vehicle C.G.
436	z_{HUB}	Vertical distance of rotor hub from vehicle C.G.
437	I_x	Inertia terms
438	I_y	
439	I_z	
440	I_{xy}	
441	I_{xz}	
442	I_{yz}	
443	I_o	
444	I_β	
445	I_ζ	
446	S_β	
447	S_ζ	
448	$I_{\beta\dot{\beta}}$	
449	$I_{\beta\alpha}$	
450	$I_{\beta\dot{\psi}}$	
451	$I_{\dot{\beta}\alpha}$	
452	$I_{\dot{\beta}\psi}$	
453	$I_{\zeta\dot{\zeta}}$	
454	$I_{\zeta\alpha}$	
455	$I_{\zeta\dot{\psi}}$	
456	$I_{\dot{\zeta}\alpha}$	
457	$I_{\dot{\zeta}\psi}$	
458	N_b	Number of blades

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
459	ν_β	Inertia terms
460	ν_ζ	
461	g_s	
462-469		
		Longitudinal Accelerometer
470	R_x	Noise covariance
471	b_x	Bias
472	k_x	Scale factor error
473	x_{cgx}	X location
474	y_{cgx}	Y location
475	z_{cgx}	Z location
		Lateral Accelerometer
479	R_y	Noise covariance
480	b_y	Bias
481	k_y	Scale factor error
482	x_{cgy}	X location
483	y_{cgy}	Y location
484	z_{cgy}	Z location
		Vertical Accelerometer
488	R_z	Noise covariance
489	b_z	Bias
490	k_z	Scale factor error
491	x_{cgz}	X location
492	y_{cgz}	Y location
493	z_{cgz}	Z location

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
		Roll Angular Accelerometer
497	$R_{\dot{p}}$	Noise covariance
498	$b_{\dot{p}}$	Bias
499	$k_{\dot{p}}$	Scale factor error
		Pitch Angular Acclerometer
503	$R_{\dot{q}}$	Noise covariance
504	$b_{\dot{q}}$	Bias
505	$k_{\dot{q}}$	Scale factor error
		Yaw Angular Accelerometer
509	$R_{\dot{r}}$	Noise covariance
510	$b_{\dot{r}}$	Bias
511	$k_{\dot{r}}$	Scale factor erro
		Roll Rate Gyro
515	R_p	Noise cavariance
516	b_p	Bias
517	k_p	Scale factor error
		Pitch Rate Gyro
521	R_q	Noise covariance
522	b_q	Bias
523	k_q	Scale factor error
		Yaw Rate Gyro
527	R_r	Noise covariance
528	b_r	Bias
529	k_r	Scale factor error

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
		Roll Position Gyro
533	R_r	Noise covariance
534	b_r	Bias
535	k_r	Scale facotr error
		Pitch Position Gyro
539	R_θ	Noise covariance
540	b_θ	Bias
541	k_θ	Scale factor error
		Yaw Position Gyro
545	R_ψ	Noise covariance
546	b_ψ	Bias
547	k_ψ	Scale factor error
		Angle-of-Attack Vane
551	R_α	Noise covariance
552	b_α	Bias
553	k_α	Scale factor error
554	$x_{cg\alpha}$	X location
555	$y_{cg\alpha}$	Y location
556	$z_{cg\alpha}$	Z location
557		
		Sideslip Vane
558	R_β	Noise covariance
559	b_β	Bias
560	k_β	Scale factor error
561	$x_{cg\beta}$	X location
562	$y_{cg\beta}$	Y location
563	$z_{cg\beta}$	Z location

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
		<u>Pitot Tube</u>
565	R_{VT}	Noise covariance
566	b_{VT}	Bias
567	k_{VT}	Scale factor error
568	x_{VT}	X location
569	y_{VT}	Y location
570	z_{VT}	Z location
571	P_o	Reference air density
572	v_s	Velocity of sound
573		
		<u>Longitudinal Velocity Measurement</u>
574	R_u	Noise covariance
575	b_u	Bias
576	k_u	Scale factor error
577		
		<u>Lateral Velocity Measurement</u>
578	R_v	Noise covariance
579	b_v	Bias
580	k_v	Scale factor error
581		
		<u>Vertical Velocity Measurement</u>
582	R_w	Noise covariance
583	b_w	Bias
584	k_w	Scale factor error
585		
586	$\tilde{x}_{ref}$	Horizontal reference distance
587		
588	$\tilde{z}_{ref}$	Vertical reference distance
589		
		<u>Rotor Longitudinal Force Measurement</u>
590	R_{xR}	Noise covariance
591	b_{xR}	Bias
592	k_{xR}	Scale factor error
593		

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
		Rotor Lateral Force Measurement
594	R_{y_R}	Noise covariance
595	b_{y_R}	Bias
596	k_{y_R}	Scale factor error
597		
		Rotor Vertical Force Measurement
598	R_{z_R}	Noise covariance
599	b_{z_R}	Bias
600	k_{z_R}	Scale factor error
601		
		Rotor Roll Moment Measurement
602	R_{L_R}	Noise covariance
603	b_{L_R}	Bias
604	k_{L_R}	Scale factor error
605		
		Rotor Pitch Moment Measurement
606	R_{M_R}	Noise covariance
607	b_{M_R}	Bias
608	k_{M_R}	Scale factor error
609		
		Rotor Yaw Moment Measurement
610	R_{N_R}	Noise covariance
611	b_{N_R}	Bias
612	k_{N_R}	Scale factor error
613		
614	ϕ_1	Euler angle
615	$\Delta\phi$	Increment
616	θ_1	Euler angle
617	$\Delta\theta$	Increment

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
618	β_o ref	Blade flapping
619	β_{1c} ref	
620	β_{1s} ref	
621		
		<u>Blade 1 Flapping Measurement</u>
622	$R_{\beta 1}$	Noise covariance
623	$b_{\beta 1}$	Bias
624	$k_{\beta 1}$	Scale factor error
625		
		<u>Blade 2 Flapping Measurement</u>
626	$R_{\beta 2}$	Noise covariance
627	$b_{\beta 2}$	Bias
628	$k_{\beta 2}$	Scale factor error
629		
		<u>Blade 3 Flapping Measurement</u>
630	$R_{\beta 3}$	Noise covariance
631	$b_{\beta 3}$	Bias
632	$k_{\beta 3}$	Scale factor error
633		
		<u>Blade 4 Flapping Measurement</u>
634	$R_{\beta 4}$	Noise covariance
635	$b_{\beta 4}$	Bias
636	$k_{\beta 4}$	Scale factor error
637		
		<u>Blade 5 Flapping Measurement</u>
638	$R_{\beta 5}$	Noise covariance
639	$b_{\beta 5}$	Bias
640	$k_{\beta 5}$	Scale factor error
641		

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
		<u>Blade 6 Flapping Measurement</u>
642	$R_{\beta 6}$	Noise covariance
643	$b_{\beta 6}$	Bias
644	$k_{\beta 6}$	Scale factor error
645		
		<u>Blade 7 Flapping Measurement</u>
646	$R_{\beta 7}$	Noise covariance
647	$b_{\beta 7}$	Bias
648	$k_{\beta 7}$	Scale factor error
649		
650	ζ_{o} ref	
651	ζ_{1c} ref	Lag references
652	ζ_{1s} ref	
653		
		<u>Blade 1 Lag Measurement</u>
654	$R_{\zeta 1}$	Noise covariance
655	$b_{\zeta 1}$	Bias
656	$k_{\zeta 1}$	Scale factor error
657		
		<u>Blade 2 Lag Measurement</u>
658	$R_{\zeta 2}$	Noise covariance
659	$b_{\zeta 2}$	Bias
660	$k_{\zeta 2}$	Scale factor error
661		
		<u>Blade 3 Lag Measurement</u>
662	$R_{\zeta 3}$	Noise covariance
663	$b_{\zeta 3}$	Bias
664	$k_{\zeta 3}$	Scale factor error
665		

Table 3.3 (Continued)

INDEX	SYMBOL	DESCRIPTION
		Blade 4 Lag Measurement
666	$R_{\zeta 4}$	Noise covariance
667	$b_{\zeta 4}$	Bias
668	$k_{\zeta 4}$	Scale factor error
669		
		Blade 5 Lag Measurement
670	$R_{\zeta 5}$	Noise covariance
671	$b_{\zeta 5}$	Bias
672	$k_{\zeta 5}$	Scale factor error
673		
		Blade 6 Lag Measurement
674	$R_{\zeta 6}$	Noise covariance
675	$b_{\zeta 6}$	Bias
676	$k_{\zeta 6}$	Scale factor error
677		
		Blade 7 Lag Measurement
678	$R_{\zeta 7}$	Noise covariance
679	$b_{\zeta 7}$	Bias
680	$k_{\zeta 7}$	Scale factor error
681		
682	$R_{S\psi_R}$	Noise covariance
683	$k_{S\psi_R}$	Scale factor error
684	$b_{S\psi_R}$	Bias
685		

Table 3.3 (Concluded)

INDEX	SYMBOL	DESCRIPTION
686	$R_{S\psi_R}$	Scale factor error
687	$k_{S\psi_R}$	Bias
688	$b_{S\psi_R}$	Noise covariance
689		
701	z_{1_o}	Reference value for expansion variable z_1
702	z_{2_o}	Reference value for expansion variable z_2
703	z_{3_o}	Reference value for expansion variable z_3
704	z_{4_o}	Reference value for expansion variable z_4
705	z_{5_o}	Reference value for expansion variable z_5

Table 3.4
Indices of the Aerodynamic Coefficients

PRIMARY SYMBOL	SUBSCRIPT																													SPARES
	o	u	v	w	p	q	r	$\dot{\beta}_0$	$\dot{\beta}_{1c}$	$\dot{\beta}_{1s}$	β_0	β_{1c}	β_{1s}	$\dot{\zeta}_0$	$\dot{\zeta}_{1c}$	$\dot{\zeta}_{1s}$	$\dot{\psi}_R$	ζ_0	ζ_{1c}	ζ_{1s}	ψ_R	θ_0	θ_{1c}	θ_{1s}	δ_{TR}	δ_e	δ_a	δ_r	δ_f	
C_X	1	2	(3)	4	(5)	6	(7)																		(8)	(9)	(10)	(11)	(12)	13-15
C_Y	16	(17)	18	(19)	20	(21)	22																		23	(24)	(25)	(26)	(27)	28-30
C_Z	31	32	(33)	34	(35)	36	(37)																		(38)	(39)	(40)	(41)	(42)	43-45
C_L	46	(47)	48	(49)	50	(51)	52																		53	(54)	(55)	(56)	(57)	58-60
C_M	61	62	(63)	64	(65)	66	(67)																		(68)	(69)	(70)	(71)	(72)	73-75
C_N	76	(77)	78	(79)	80	(81)	82																		83	(84)	(85)	(86)	(87)	88-90
H	91							92	93	94	95	96	97	98	99	100	101	102	103	(104)	(105)	106	107	108						109-110
T	111							112	113	114	115	116	117	118	119	120	121	122	123	(124)	(125)	126	127	128						129-130
Y	131							132	133	134	135	136	137	138	139	140	141	142	143	(144)	(145)	146	147	148						149-150
L	151							152	153	154	155	156	157	158	159	160	161	162	163	(164)	(165)	166	167	168						169-170
M	171							172	173	174	175	176	177	178	179	180	181	182	183	(184)	(185)	186	187	188						189-190
Q	191							192	193	194	195	196	197	198	199	200	201	202	203	(204)	(205)	206	207	208						209-210
H^0	211	212	213	214	215	216	217	281	219	220	221	222	223	224	225	226	227	228	229	(230)	(231)	232	233	234						235
H^{1c}	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	(255)	(256)	257	258	259						260
H^{1s}	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	(280)	(281)	282	283	284						285
Q^0	286	287	288	289	290	291	292	293	244	295	296	297	298	299	300	301	302	303	304	(305)	(306)	307	308	309						310
Q^{1c}	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	(330)	(331)	332	333	334						335
Q^{1s}	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	(355)	(356)	357	358	359						360
Q ...	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	(380)	(381)	382	383	384						385-400

```
CH-53A NONLINEAR INPUT DESIGN RUN  100 KNOTS
  5    1   14   10    5
  2
5.0                  .01       0.        0.        .05
 500  .01            0
   1    5    5    1    3
0.
.1
1.
          1.
                    1.
                              1.
                                        1.
.26224    .24048    .34029    .00031554 .00030634 .00007458 .0000767957.5433E-05
3.3348E-064.4698E-05
0.
0.
100.
    4    0    0    0    0    2
    1    1   50  500
STATES    U    V    W    P    Q    R    PHI  THET B0   B0D  BC   BCD  BS   BSD
MEASURE   AX   AY   AZ   QD   RD   P    Q    R    PHI  THET
EXP VAR   DTR
*
 426 G        32.174
 427 RHO      .0023769
 428 THETAR   0.
 429 OMEGA    19.33
 430 R        36.
 431 SIGMA    0.1151
 432 GAMMA    10.84
 433 CLA      6.09
 434 MASS     1041.
 435 XHUB     0.
 436 ZHUB     0.
 437 IX       41400.
 438 IY       206500.
```

Figure 3.1 Listing of Input Data

439	IZ	192500.	
442	IYZ	0.	
443	IO	4865.	
444	IBETA	4865.	
458	NB	6.	
489	AZBIAS	32.174	
470	RAX	.26224	
479	RAY	.24048	
488	RAZ	.34029	
503	RQDOT	.00031554	
509	RRDOT	.00030634	
515	RP	.000074580	
521	RQ	.000076795	
527	RR	.000075433	
533	RPHI	.0000033346	
539	RTHETA	.0000044698	
1	CXO		1
2	CXU	-.000742	2
3	CXV	+.0000649	3
4	CXW	+.0146	4
5	CXP	-.000541	5
6	CXQ	-.0428	6
7	CXR	+.00371	7
8	CXDTR	-.000107	8
9	CYO	+.0	16
10	CYU	+.00241	17
11	CYV	-.00876	18
12	CYW	+.000334	19
13	CYP	+.0285	20
14	CYQ	-.00293	21
15	CYR	-0.717	22
16	CYDTR	+.00148	23
17	CZO	+.0	31
18	CZU	-.0192	32
19	CZV	-.0011	33
20	CZW	-.115	34
21	CZP	-.00263	35

Figure 3.1 (Continued)

22	CZQ	+0.829	36
23	CZR	+.0189	37
24	CZDTR	.000119	38
25*	CLO	.0	46
26	CLU	-.000138	47
27	CLV	-.0027	48
28	CLW	-.00159	49
29*	CLP	-.000816	50
30	CLQ	.0000806	51
31	CLR	.0021	52
32	CLDTR	.000945	53
33	CMO	.00	61
34	CMU	-.000426	62
35	CMV	.00196	63
36	CMW	.00135	64
37	CMP	-.00013	65
38	CMQ	-.1183	66
39	CMR	.000106	67
40	CMDTR	.0000096	68
41	CNO	.0	76
42	CNU	-.00127	77
43	CNV	.00643	78
44	CNW	-.000159	79
45	CNP	.00245	80
46	CNQ	.00131	81
47	CNR	-.0181	82
48	CNDTR	-.0059	83
49	HO	.0	91
50	HBO	-.00558	95
51	HBOD	-.000179	92
52	HBC	.00533	96
53	HBCD	.0103	93
54	HBS	.000424	97
55	HBSD	.0021	94
56	HTS	.00109	108
57	HTC	.00109	107
58	HTO	-.00589	106

Figure 3.1 (Continued)

59	YO	.0	131
60	YBO	-.00224	135
61	YBOD	-.00151	132
62	YBC	.00096	136
63	YBCD	-.000902	133
64	YBS	.00573	137
65	YBSD	.00398	134
66	YTS	.00257	148
67	YTC	.00254	147
68	YTO	-.000157	146
69	TO	.007147	111
70	TBO	.0484	115
71	TBOD	-.00343	112
72	TBC	.00247	116
73	TBCD	-.0162	113
74	TBS	-.0025	117
75	TBSD	.0193	114
76	TTS	-.0275	128
77	TTC	-.0038	127
78	TTO	.0598	126
79	LO	.0	151
80	LBO	.000128	155
81	LBOD	.0000474	152
82	LBC	.00112	156
83	LBCD	.00224	153
84*	LBS	-.00437	157
85	LBSD	-.00508	154
86	LTS	-.000224	168
87*	LTC	-.000817	167
88	LTO	.000847	166
89	MO	.0	171
90	MBO	-.0000783	175
91	MBOD	-.000996	172
92	MBC	.0047	176
93	MBCD	.0071	173
94	MBS	.00122	177
95	MBSD	.00245	174

Figure 3.1 (Continued)

96	MTS	-.000431	188	
97	MTC	-.0000973	187	
98	MTO	.000508	186	
99	QO	.0	191	
100	QBO	.0016	195	
101	QBOD	.000335	192	
102	QBC	.000947	196	
103	QBCD	.00117	193	
104	QBS	-.0021	197	
105	QBSD	-.00176	194	
106	QTS	.000103	208	
107	QTC	.00052	207	
108	QTO	-.0536	206	
109	BOO	.0	211	
110	BOU	.00057	212	
111	BOV	.00001	213	
112	BOW	.00867	214	
113	BOP	.000276	215	
114	BOQ	-.00221	216	
115	BOR	-.00221	217	
116	BOBO	-.00484	221	
117	BOBOD	-.00734	218	
118	BOBC	.000542	222	
119	BOBCD	.000557	219	
120	BOBS	-.000484	223	
121	BOBSD	-.000325	220	
122	BOTS	-.00165	234	
123	BOTC	.000283	233	
124	BODTR	-.0000559	211	1
125	BOTO	.00656	232	
126	BCO	.0	236	
127	BCU	.00179	237	
128	BCV	-.00142	238	
129	BCW	.000761	239	
130	BCP	-.00472	240	
131	BCQ	-.0105	241	
132	BCR	.000037	242	

Figure 3.1 (Continued)

```
 133 BCBO      .00258            246
 134 BCBOD     .00155            243
 135 BCBC     -.00138            247
 136 BCBCD    -.0108             244
 137 BCBS     -.00454            248
 138 BCBSD    -.0132             245
 139 BCTS     -.00337            259
 140 BCTC      .0747             258
 141 BCDTR    -.0000153          236 1
 142 BCTO      .000385           257
 143 BSO       .0                261
 144 BSU      -.00129            262
 145 BSV      -.00102            263
 146 BSW      -.0019             264
 147 BSP      -.00847            265
 148 BSQ       .00674            266
 149 BSR      -.000401           267
 150 BSBO     -.00024            271
 151 BSBOD     .00037            268
 152 BSBC      .00444            272
 153 BSBCD     .0127             269
 154 BSBS     -.0014             273
 155 BSBSD    -.00981            270
 156 BSTS      .0047             284
 157*BSTC      .00317            283
 158 BSDTR     .0000084          261 1
 159 BSTO     -.00242            282
*
Y( 1) LONG ACCEL  (FT/SEC**2)
Y( 2) KAT ACCEL   (FT/SEC**2)
Y( 3) VERT ACCEL  (FT/SEC**2)
*
```

Figure 3.1 (Concluded)

IV. PROGRAM OUTPUT

The sample input deck in Figure 3.1 was run on INDES. Some of the resulting output will be shown in this section. The rest of the output is keyed to the discussion in Section 2.2. The summary page is shown in Figure 4.1. The namelist input variables are listed in Figure 4.2. The array defining one portion of the control input is shown in Figure 4.3. Figure 4.4 shows the beginning of the output of the optimal control sequence computed by INDES, while Figure 4.5 is an excerpt from the sequence at a later time. Figure 4.6 is a sample of the plots which come out on the printer.

```
CH-53A NONLINEAR INPUT DESIGN RUN  100 KNOTS

*****************************************************************
*                                                               *
*                    INPUT DESIGN PROGRAM                       *
*                  -------------------------                    *
*                                                               *
*                                                               *
*                                      WRITTEN BY:              *
*                                      SYSTEMS CONTROL, INC.    *
*                                      PALO ALTO, CAL.          *
*                                                               *
*****************************************************************

THE PROGRAM WAS SET TO THE FOLLOWING DIMENSIONS...
                                                HAS LIMIT EXCEEDED?
  14  STATES . . . . . . . . . . . . . .              NO
  10  MEASUREMENTS . . . . . . . . . . .              NO
   1  CONTROLS . . . . . . . . . . . . .              NO
 722  PARAMETERS IN MODEL . . . . . . .               NO
 500  DATA POINTS IN TIME HISTORIES . .               NO
   5  PARAMETERS TO BE IDENTIFIED


   5 TIME INTERVALS
     DELTA T  =      .0100
     TOTAL TIME =      5.0000 SECONDS


 DESIGN FOR CASE   1
 WITH LIMIT ON I = 5
 AND LIMIT ON J  = 5

EXPANSION VARIABLES USED IN THIS RUN...
  Z(1) = TAIL ROTOR

THE FLAG ARRAYS FOR THIS RUN ARE...
  ISFLAG =    1  2  3  4  5  6  7  8  0 10 12 14  9 11
  NSFLAG =    1  2  3  4  5  6  7  8 13 10 14 11 15 12
  NMFLAG =    1  2  3  5  6  7  8  9 10 11
  NZFLAG =    9
```

Figure 4.1 Summary Page

```
$DOUT
NS     = 14,
NQ     = 10,
NP     = 5,
NDUM   = 1,
NEXV   = 5,
NPEX   = 400,
MAXP   = 722,
MMP    = 5,
NW     = 500,
NSA    = 84,
NPA    = 60,
NSQ    = 0,
LCV    = 20,
NXA    = 0, 1,
NYA    = 60, 1,
NCV    = 15, 15,
NR     = 5, 5,
NIQ    = 1,
$END
```

Figure 4.2 Namelist Input

CONTROL NO. 1

0.	.10000E-01	0.	0.	0.			
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

Figure 4.3 Control Input

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	[illegible]	[illegible]	[illegible]	[illegible]

Figure 4.3 (Continued)

```
SYSTEM SIMULATION WITH OPTIMAL INPUTS

TIME(SEC)     CONTROLS--                                               MEASUREMENTS--

   0.000        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.          1.92962E+01 9.12043E-01 1.31261E+00-6.10039E-03
                                                                       -4.16990E-02 0.          0.          0.          0.
                                                                        0.
    .010        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.          1.44337E+01 9.95391E-01 7.18085E+00 5.78937E-01
                                                                       -1.19187E-01-7.35428E-04 2.90210E-03-8.39549E-04 1.01765E-06
                                                                        9.63108E-06
    .020        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.          1.00540E+01 1.69708E+00 1.09633E+01 1.10540E+00
                                                                       -1.54580E-01-4.39014E-03 1.13825E-02-2.24311E-03-2.43756E-05
                                                                        7.66638E-05
    .030        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.          6.30126E+00 2.91477E+00 1.31547E+01 1.55295E+00
                                                                       -1.49672E-01-5.95292E-03 2.47462E-02-3.79655E-03-7.97371E-05
                                                                        2.53575E-04
    .040        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.          3.25545E+00 4.52859E+00 1.42235E+01 1.90975E+00
                                                                       -1.08584E-01-1.18764E-03 4.21382E-02-5.11576E-03-1.22272E-04
                                                                        5.85018E-04
    .050        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.          9.41579E-01 6.41225E+00 1.45931E+01 2.17137E+00
                                                                       -3.71839E-02 1.32657E-02 6.26232E-02-5.86707E-03-7.11612E-05
                                                                        1.10664E-03
    .060        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.         -6.60620E-01 8.44229E+00 1.46294E+01 2.33955E+00
                                                                        5.75445E-02 3.98591E-02 8.52535E-02-5.78162E-03 1.83476E-04
                                                                        1.84462E-03
    .070        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.         -1.60645E+00 1.05054E+01 1.46341E+01 2.42088E+00
                                                                        1.68124E-01 8.01615E-02 1.09124E-01-4.66336E-03 7.71602E-04
                                                                        2.81586E-03
    .080        0.          7.67411E-02 0.          0.                  0.          0.          0.          0.          0.
                                                                        0.         -1.97605E+00 1.25034E+01 1.48419E+01 2.42547E+00
                                                                        2.87167E-01 1.34927E-01 1.33415E-01-2.39100E-03 1.83476E-03
                                                                        4.02856E-03
```

Figure 4.4 Beginning of Optimal Control Sequence

```
.090   0.   7.67411E-02 0.   0.     0.          0.          0.          0.          0.
                                    0.         -1.86516E+00 1.43565E+01 1.54223E+01 2.36574E+00
                                    4.07840E-01 2.04196E-01 1.57419E-01 1.08529E-03 3.51843E-03
                                    5.48324E-03
.100   0.   7.67411E-02 0.   0.     0.          0.          0.          0.          0.
                                    0.         -1.37669E+00 1.60047E+01 1.64842E+01 2.25528E+00
                                    5.24231E-01 2.87411E-01 1.80561E-01 5.75139E-03 5.96547E-03
                                    7.17388E-03
.110   0.   7.67411E-02 0.   0.     0.          0.          0.          0.          0.
                                    0.         -6.13813E-01 1.74073E+01 1.80829E+01 2.10786E+00
                                    6.31557E-01 3.83549E-01 2.02402E-01 1.15395E-02 9.31084E-03
                                    9.08920E-03
.120   0.   7.67411E-02 0.   0.     0.          0.          0.          0.          0.
                                    0.          3.25726E-01 1.85418E+01 2.02280E+01 1.93669E+00
                                    7.26264E-01 4.91246E-01 2.22639E-01 1.83403E-02 1.36776E-02
                                    1.12140E-02
.130   0.   7.67411E-02 0.   0.     0.          0.          0.          0.          0.
                                    0.          1.35372E+00 1.94020E+01 2.28923E+01 1.75376E+00
                                    8.06000E-01 6.08920E-01 2.41097E-01 2.60148E-02 1.91740E-02
                                    1.35302E-02
.140   0.   7.67411E-02 0.   0.     0.
```

Figure 4.4 (Continued)

```
                                                                       -7.01216E-02 8.35531E+00 1.28425E-01-4.65829E-01 1.25442E+01
                                                                       -1.14385E-01
1.960     0.          5.99032E-02 0.          0.                        0.           0.           0.           0.           0.
                                                                        0.          -7.19723E-01 1.43961E+02-2.54742E+01 1.03829E-01
                                                                       -5.99085E-02 8.34895E+00 1.29400E-01-4.66479E-01 1.26282E+01
                                                                       -1.13004E-01
1.970     0.          5.99032E-02 0.          0.                        0.           0.           0.           0.           0.
                                                                        0.          -5.35595E-01 1.43869E+02-2.69518E+01 1.15549E-01
                                                                       -4.91715E-02 8.34426E+00 1.30498E-01-4.67026E-01 1.27122E+01
                                                                       -1.11229E-01
1.980     0.         -1.65025E-03 0.          0.                        0.           0.           0.           0.           0.
                                                                        0.           1.88362E+00 1.43056E+02-3.16539E+01-1.03975E-01
                                                                        3.36076E-02 8.33939E+00 1.30948E-01-4.67171E-01 1.27961E+01
                                                                       -1.09071E-01
1.990     0.         -1.65025E-03 0.          0.                        0.           0.           0.           0.           0.
                                                                        0.           5.72412E+00 1.42591E+02-3.63323E+01-5.35992E-01
                                                                        1.21710E-01 8.33665E+00 1.27707E-01-4.66392E-01 1.28799E+01
                                                                       -1.06570E-01
2.000     0.         -1.65025E-03 1.92962E+01 9.12043E-01              1.31261E+00-6.10039E-03-4.16990E-02 0.           0.
                                                                        0.           9.07995E+00 1.41584E+02-3.91788E+01-9.09323E-01
                                                                        2.05519E-01 8.33417E+00 1.20425E-01-4.64751E-01 1.29637E+01
                                                                       -1.03784E-01
2.010     0.          0.          1.44337E+01 9.95391E-01              7.18085E+00 5.78937E-01-1.19187E-01-7.35428E-04 2.90210E-03
                                                                       -8.39549E-04 1.18033E+01 1.40095E+02-4.04819E+01-1.20442E+00
                                                                        2.80930E-01 8.32829E+00 1.09782E-01-4.62310E-01 1.30474E+01
                                                                       -1.00770E-01
2.020     1.01765E-06 9.63108E-06 1.00540E+01 1.69708E+00              1.09633E+01 1.10540E+00-1.54580E-01-4.39014E-03 1.13825E-02
                                                                       -2.24311E-03 1.38712E+01 1.38163E+02-4.06713E+01-1.41493E+00
                                                                        3.47698E-01 8.31592E+00 9.66196E-02-4.59160E-01 1.31310E+01
                                                                       -9.75740E-02
2.030    -2.43756E-05 7.66638E-05 6.30126E+00 2.91477E+00              1.31547E+01 1.55295E+00-1.49572E-01-5.95292E-03 2.47462E-02
                                                                       -3.79655E-03 1.53326E+01 1.35889E+02-4.01053E+01-1.54722E+00
                                                                        4.04664E-01 8.29482E+00 8.17446E-02-4.55389E-01 1.32143E+01
                                                                       -9.42349E-02
2.040    -7.97371E-05 2.53575E-04 3.25545E+00 4.52859E+00              1.42235E+01 1.90975E+00-1.08584E-01-1.18764E-03 4.21382E-02
                                                                       -5.11576E-03 1.62150E+01 1.33356E+02-3.90314E+01-1.60487E+00
                                                                        4.50778E-01 8.26354E+00 6.59241E-02-4.51103E-01 1.32974E+01
                                                                       -9.07817E-02
2.050    -1.22272E-04 5.85018E-04 9.41579E-01 6.41225E+00              1.45931E+01 2.17137E+00-3.71839E-02 1.32657E-02 6.26232E-02
                                                                       -5.86707E-03 1.65675E+01 1.30646E+02-3.76497E+01-1.59460E+00
                                                                        4.85412E-01 8.22124E+00 4.98734E-02-4.46412E-01 1.33801E+01
                                                                       -8.72356E-02
```

Figure 4.5 Later in the Optimal Control Sequence

```
2.060    -7.11512E-05 1.10664E-03-6.60620E-01 8.44229E+00     1.46294E+01 2.33955E+00 5.75445E-02 3.98591E-02 8.52535E-02
                                                            -5.78162E-03 1.64524E+01 1.27840E+02-3.61122E+01-1.52525E+00
                                                             5.08327E-01 8.16773E+00 3.42291E-02-4.41434E-01 1.34623E+01
                                                            -8.36110E-02
2.070     1.83476E-04 1.84462E-03-1.60645E+00 1.05054E+01     1.46341E+01 2.42088E+00 1.68124E-01 8.01615E-02 1.09124E-01
                                                            -4.66336E-03 1.59410E+01 1.25013E+02-3.45277E+01-1.40696E+00
                                                             5.19719E-01 8.10329E+00 1.95315E-02-4.36284E-01 1.35438E+01
                                                            -7.99171E-02
2.080     7.71602E-04 2.81586E-03-1.97605E+00 1.25034E+01     1.48419E+01 2.42547E+00 2.87167E-01 1.34927E-01 1.33415E-01
                                                            -2.39100E-03 1.51069E+01 1.22233E+02-3.29654E+01-1.25030E+00
                                                             5.20158E-01 8.02865E+00 6.21758E-03-4.31076E-01 1.36247E+01
                                                            -7.61599E-02
2.090     1.83476E-03 4.02856E-03-1.86516E+00 1.43565E+01     1.54223E+01 2.36574E+00 4.07840E-01 2.04196E-01 1.57419E-01
                                                             1.08529E-03 1.40229E+01 1.19557E+02-3.14622E+01-1.06574E+00
                                                             5.10584E-01 7.94484E+00-5.38157E-03-4.25914E-01 1.37047E+01
                                                            -7.23436E-02
2.100     3.51843E-03 5.48324E-03-1.37669E+00 1.60047E+01     1.64842E+01 2.25528E+00 5.24231E-01 2.87411E-01 1.80561E-01
                                                             5.75139E-03 1.27575E+01 1.17034E+02-3.00271E+01-8.63163E-01
                                                             4.92200E-01 7.85315E+00-1.50373E-02-4.20894E-01 1.37838E+01
                                                            -6.84725E-02
2.110     5.96547E-03 7.17388E-03-6.13813E-01 1.74073E+01     1.80829E+01 2.10786E+00 6.31557E-01 3.83549E-01 2.02402E-01
                                                             1.15395E-02 1.13728E+01 1.14700E+02-2.86493E+01-6.51484E-01
                                                             4.66430E-01 7.75501E+00-2.26148E-02-4.16095E-01 1.34[illegible]
```

Figure 4.5 (Continued)

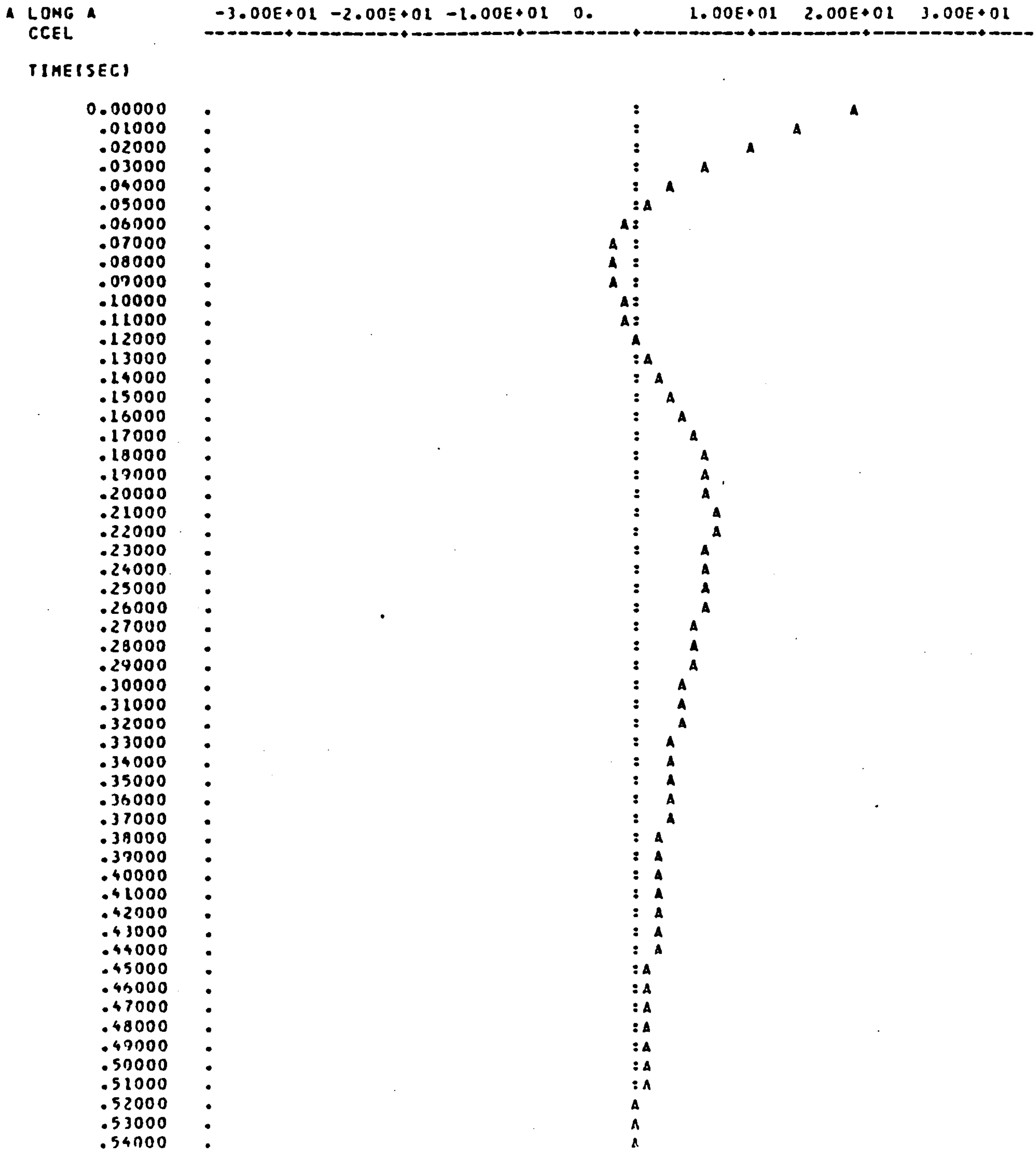


Figure 4.6 Sample of Printer Plots